Coupled FEM-BEM Analysis of the TEAM-7 Problem

FEM-BEM COUPLING

In the numerical analysis of electromagnetic field problems, one is often confronted with complex three-dimensional geometries, moving parts and unbounded regions. Although the finite element method (FEM) has been successfully applied to many problems incorporating nonlinear material behavior in the solid domains, its application to the unbounded air region typically induces an artificial truncation of the analysis domain which incorporates a modeling error. Moreover, the handling of moving parts with a pure FEM approach is very involved and lacks general applicability. As a remedy, we propose to apply a boundary element method (BEM) to the unbounded air region which only requires a surface mesh. Together with FEM for the solid domains, a FEM-BEM coupling approach is proposed which combines the respective advantages of each method.

FEATURES

The FEM-BEM coupling software provides the following functionality:

- Nédélec finite elements of first and second order
- Implementation with tetrahedron, hexahedron, prism and pyramid elements
- Handling of non-linear *B*-*H* curves
- Symmetric Galerkin BEM with robust integration routines
- Multilevel Fast Multipole Method for the realization of the matrix-vector products
- Unconditionally stable monolithic coupling
- Optimal preconditioners for FEM and BEM parts
- Automatic treatment of non-simply connected domains

TECHNICAL DETAILS

Eddy current problems form an important sub-class of electromagnetic field problems. Using a vector potential $\mathbf{A}$ with $\mathbf{B} = \mathbf{curl}\,\mathbf{A}$, the variational formulation of the eddy current problem is: Find $\mathbf{A} \in H(\mathbf{curl})$ such that

$$\langle \mu^{-1}\,\mathbf{curl}\,\mathbf{A}, \mathbf{curl}\,\mathbf{A}' \rangle_\Omega + \langle \sigma \tfrac{\partial \mathbf{A}}{\partial t}, \mathbf{A}' \rangle_\Omega - \langle \mathbf{n} \times \mathbf{curl}\,\mathbf{A}, \mathbf{n} \times \mathbf{A}' \times \mathbf{n} \rangle_\Gamma = \langle \mathbf{j}, \mathbf{A}' \rangle_\Omega$$

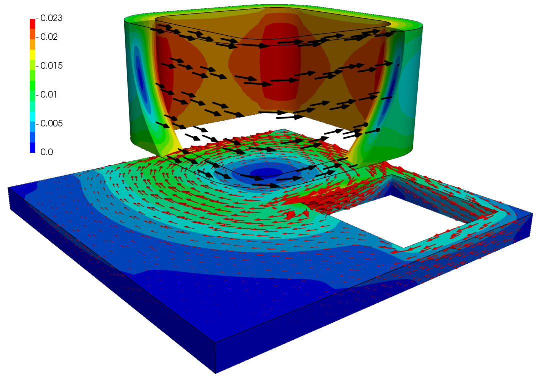
for all test-functions $\mathbf{A}' \in H(\mathbf{curl})$. The domain of interest is denoted as Ω and Γ is its boundary with outward normal $\mathbf{n}$. The right hand side is given by some excitation source current $\mathbf{j}$.

FEM formulations are obtained by a proper discretization of the volume terms $\langle \cdot, \cdot \rangle_\Omega$ and they neglect the boundary term $\langle \cdot, \cdot \rangle_\Gamma$. However, a coupled FEM-BEM formulation utilizes this term by help of Boundary Integral operators. With this approach[1] the overall numerical scheme becomes a little bit more complex but the reward is obvious:

- No arbitrarily large volume discretization of the unbounded, surrounding air-region has to be exploited.
- No modeling error occurs. The decay conditions of $\mathbf{A}$ and $\mathbf{B}$ are fulfilled exactly.
- Models with moving parts can be easily tackled.

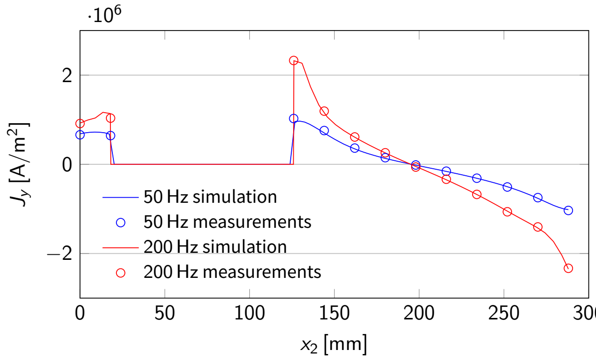
[1] See https://doi.org/10.1108/COMPEL-02-2017-0061 for details.

VERIFICATION



Magnitude of Re($\mathbf{B}$) with applied current (black) and induced current (red) for 200 Hz

For verification purposes, problem 7 of the TEAM benchmarks is considered and a frequency-domain simulation with an alternating current excitation is carried out. For the analysis, 56 600 finite elements are used which leads to approximately 160K FEM and 34K BEM degrees of freedom.



Comparison of induced currents for the frequencies 50 Hz and 200 Hz along the line $0 \leq x \leq 288$ mm, $y = 72$ mm, $z = 19$ mm

The evaluation of the y-component of the induced currents along a given line show an excellent agreement with published measurement data. Moreover, owed to the Fast Multipole Method and optimal preconditioning techniques the computational costs is linearly proportional to the system size.

A typical application for these kind of problems are eddy-current brakes. An illustrative example can be found here: http://tailsit.com/lenz/lenz.gif.

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